**And Gate Probability**

When a top event (i.e. catastrophic system failure, explosion, etc.) is the result of contributing events connected through an **AND** gate, the total probability of occurrence is somewhat less than if those events passed through an **OR** gate. Under an **AND** gate, each listed event must occur in order to realize the top event. Under an **OR** gate, only one of many possible events must occur for the top or main event to occur. Probable occurrence of an event supported by an **AND** gate requires multiplication of the underlying events probabilities.

For example, if an explosion occurred as a result of a valve failure (top event) is supported through an **AND** gate by three contributing events entitled operator error, valve malfunction, and system over pressurization, the simple fault tree could be drawn as shown below:

![Fault Tree Diagram](image)

Manufacturer provided valve malfunction probabilities to be .025 based on 1000 h (hours) of operation (25 failures in every 1000 h). Past accident/incident experience has determined the probability for operator error to be .031 (31 errors in 1000 h of performance). Similar historical experience has shown that the probability of a system over pressurization is very unlikely and the probability is .001 (1 in a 1000 h of operation).

The probability of an explosion could be presented as

\[
P(A) = P(B) \cdot P(C) \cdot P(D)
\]

where \( P(A) \) is the probability of an explosion

\( P(B) \) is the probability of valve malfunction

\( P(C) \) is the probability of operator error

\( P(D) \) is the probability of system over pressurization

**Or Gate Probability**

When occurrence of the top event is not dependent on the occurrence of all subevents, the probability of the top event increases. That requires a more informed decision regarding hazard risk acceptance.

If the analysis determined that an occurrence of any or all of the three events would result in simple system failure and that these events were mutually exclusive, then these events would be connected to the top event through an **OR** gate and the probability values should be added, according to the formula below:

\[
P(A) = P(B) + P(C) + P(D)
\]
The Poisson probability formula uses expected number of mishaps based on previous experience.

\[ P_X = \frac{e^{-M} M^X}{X!} \]

Where:
- \( M = NP \)
- \( e = 2.178 \)
- \( N \) = Exposure period
- \( P \) = Probability of one mishap
- \( X \) = Number of mishaps in question
- \( M \) = Expected number of mishaps

In mathematics, the **factorial** of a non-negative integer \( n \) or \( X \) (in our case), denoted by \( n! \), is the product of all positive integers less than or equal to \( n \). For example,

\[
5! = 1 \times 2 \times 3 \times 4 \times 5 = 120 \\
6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720
\]

where \( n! \) represents \( n \) factorial. The notation \( n! \) was introduced by Christian Kramp in 1808.

**Example:**
A ventilation system is used in a manufacturing setting to reduce particulate matter exposure. Based on manufacturers’ data, the HVAC unit had 4 failures after 45,000 hours of operation. The manufacturer offers a new system that had significant improvements intended to reduce the number of failures. The new system was run for 75,000 hours, during which time it had only 2 failures.

By multiplying the number of exposure units (\( N \)) by the probability of 1 failure (\( P \)), we can find \( M \) which is the expected number of failures. The exposure time is 75,000 hours.

The new exposure time is 75,000 hours, and the probability of 1 failure is \( 8.8 \times 10^{-5} \). During the 75,000 hours, there would be 6.6 failures. \( (8.8 \times 10^{-5} \times 75,000) \) based on previous conditions (4 failures after 45,000 hours of operation \( 4/45,000=0.000088 \)). The term \( e \) is the base of a natural logarithm and has value 2.178, and the variable \( X \) represents the specific number of failures being investigated. In this example \( X=2 \).
Conditional Probability

\[ P_{A/B} = \frac{P_{A\text{and}B}}{P_A} \]

Example
A safety manager wanted to determine the probability of having a lost workday charged a worker because of back injury. The safety manager determined that for the 87 claims of all types reported in one year, 18 involved lost workdays. Of the 87 reported claims, 23 were for back injuries, and of these 23 cases, 14 resulted in lost workdays. The safety manager wishes to determine the probability of a lost workday case if the case was the result of back injury.

\[ P_{LWD/BI} = \frac{P_{LWD\&BI}}{P_{BI}} \]